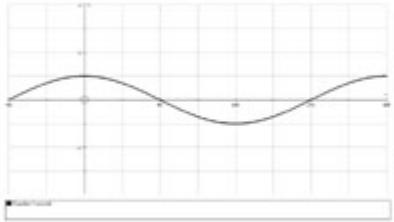
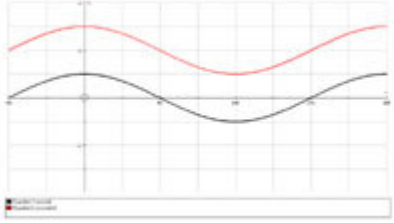
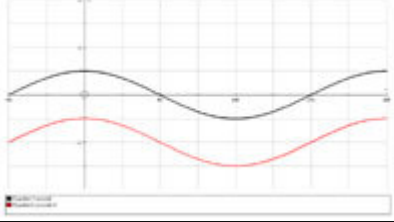
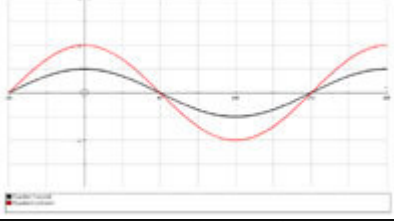
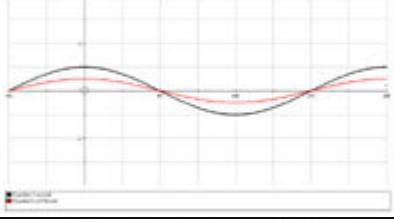
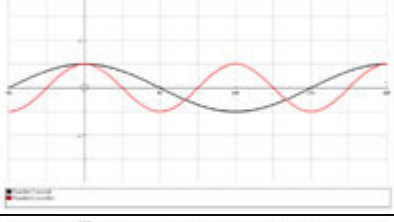

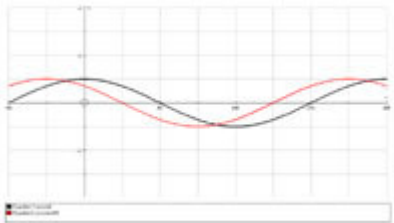
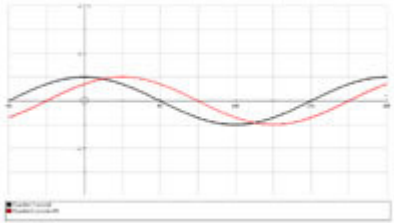
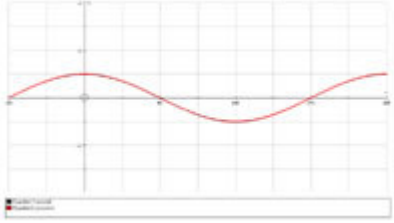
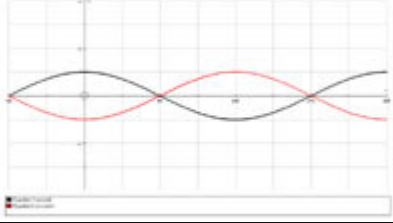


The Effect of Transformations on the Graph of a Function

When the original graph of $f(x)$ is transformed using the constant a it has the following effects, illustrated with the graph of $f(x)=\cos(x)$, which is always black. The transformation is always red.

$f(x)$			
$f(x) + a$	$\uparrow a$		<ul style="list-style-type: none"> • Curve is moved upwards • Amplitude same • Period same
$f(x) - a$	$\downarrow a$		<ul style="list-style-type: none"> • Curve is moved downwards • Amplitude same • Period same
$af(x)$	$\updownarrow a$		<ul style="list-style-type: none"> • Curve is stretched in y direction • Amplitude multiplied by a • Period same
$\frac{1}{a}f(x)$	$\downarrow \uparrow a$		<ul style="list-style-type: none"> • Curve is squashed in y direction • Amplitude divided by a • Period same
$f(ax)$	$\rightarrow\leftarrow a$		<ul style="list-style-type: none"> • Curve is squashed in the x direction • Amplitude same • Period divided by a
$f\left(\frac{1}{a}x\right)$	$\leftarrow\rightarrow a$		<ul style="list-style-type: none"> • Curve is expanded in the x direction • Amplitude same • Period multiplied by a

$f(x + a)$	$\leftarrow a$		<ul style="list-style-type: none"> • Curve is shifted left by a • Amplitude same • Period same
$f(x - a)$	$\rightarrow a$		<ul style="list-style-type: none"> • Curve is shifted right by a • Amplitude same • Period same
$f(-x)$	reflection in y axis		<ul style="list-style-type: none"> • Curve is reflected in y axis • Amplitude same • Period same <p>(on reflection it may have been better to use $f(x)=\sin(x)$ to illustrate - why?)</p>
$-f(x)$	reflection in x axis		<ul style="list-style-type: none"> • Curve is reflected in x axis • Amplitude same • Period same

To help you remember:

- 'a' outside the brackets multiplied: does stretch you expect, in the y direction
- 'a' outside the brackets added: does translation you expect, in the y direction
- 'a' inside the brackets multiplied: does opposite stretch to what you expect in the x direction
- 'a' inside the brackets added: does opposite translation to what you expect in the x direction